Big Bang Nucleosynthesis

The emergence of elements in the universe

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Abstract.

In this paper, I will first give a brief overview of what general relativity has to say about cosmology, getting an expanding universe as a solution to Einstein's equation, i.e. a universe with a [thermal] history. We will go through the different steps of the big bang nucleosynthesis, briefly justifying the particle-antiparticle asymmetry (otherwise no nucleosynthesis would happen) and then evaluating and discussing in details the abundances of the first elements.
I will then discuss the consequences of the Big Bang B nucleosynthesis on modern physics: the constraints it gives on the standard model, on dark matter...

Contents

Disclaimer.
This lecture is mainly based on Jean Philippe Uzan and Patrick Peter's book "Cosmologie Primordiale" (chapter4), Mark Trodden and Sean Carroll's "TASI Lectures : Introduction to Cosmology", and the review article "Big Bang nucleosynthesis and physics beyond the Standard Model".
The complete list of references used can be found on the last page of this document.

Introduction to standard cosmology

The first cosmological solutions to Einstein's equations were given by Einstein himself in the early 1917. However the general solutions were independently found by Alexandre Friedmann and Georges Lemaitre only in 1922 and 1927.

The standard Big Bang cosmological model is based on what is now called the "Cosmological Principle", which assumes that the universe is spatially homogeneous and isotropic. This principle enforces the geometry of the universe to be one that is described by Friedmann and Lemaitre solutions to Einstein's equations.

a) The Friedmann-Lemaitre-Robertson-Walker models

The symmetries induced by homogeneity and isotropy of space allow us to write the metric in a very simple and elegant form:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - R^2(t)[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin\theta^2\, d\phi^2)\,]$$

Here R(t) is the cosmic scale factor which evolves in time and describes the expansion –or contraction- of the universe and k is the scaled 3-space curvature signature (+1=elliptic, 0=euclidean,-1=hyperbolic space ; it is an information on the local geometry of the universe).

Another useful quantity to define from the cosmic scale factor is the Hubble parameter given by:

$$H = \frac{\dot{R}}{R}$$

The Hubble parameter is the measure of the expansion rate of the universe (it is an expansion rate because it is homogenous to an inverse time: $[H] = T^{-1}$) which links the recession speed of a galaxy *v* to its distance *d* through the following law, known as the Hubble law, $v \approx Hd$.

Coming back to Einstein's full field equation $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu}$, the question we first need to ask ourselves is what kind of energy-momentum tensor can be consistent with observations and the cosmological principle. It turns out that it is often useful (and simpler) to consider the matter of the universe as a perfect fluid:

$$T_{\mu\nu} = \rho g_{00} + p g_{ii}$$

With ρ the energy density in the rest frame of the fluid and p its pressure in the same frame, g_{ij} being the spatial metric (including the R^2).

Plugging it into Einstein's equations, we get the two following equations, the first one being an evolution equation and the second one a constraint to it:

$$\frac{\ddot{R}}{R}+\frac{1}{2}H^2=-4\pi Gp-\frac{k}{2R^2}+\frac{\Lambda}{6}$$

$$H^2=\frac{8\pi G}{3}\rho-\frac{k}{R^2}+\frac{\Lambda}{3}$$

We therefore get a dynamical universe, to which we can give an age:

$$\tau\sim\frac{1}{H}=14.2\ Gyr$$

and we can now turn ourselves to understanding its [thermal] evolution, or in other words its thermodynamics.

b) Thermal history of the universe

From very basic arguments, we can understand that the composition of the universe has greatly evolved; let's call Γ reaction rate for a given particle interaction. If that reaction rate is much higher than the expansion rate H, then the involved interaction can maintain those particles in a thermodynamic equilibrium at a temperature T; they can then be treated as Fermi-Dirac or Bose-Einstein gases, obeying the following distribution function:

$$f_i(E,T)=\frac{g_i}{e^{[\frac{E-\mu_i}{kT_i(t)}]}\pm 1}$$

With g_i the degeneracy factor, μ_i is the chemical potential, $E^2=p^2+m^2$, and $T_i(t)$ is the temperature of the kind of particle studied. The temperature of the photons, T, is called temperature of the universe.

However, if $\Gamma<H$ the particle is said to be decoupled; the interaction can maintain the thermodynamic equilibrium between the particle and the other constituents.
From this we understand that there always exists a temperature for which the interaction is not effective anymore; it is said to be "frozen".

The particle density equation can be computed from the distribution function:

$$n_i(t)=\int_m^\infty\frac{\sqrt{E^2-m^2}}{e^{\frac{E-\mu_i}{kT_i(t)}}\pm 1}EdE$$

This equation, applied to bosons and fermions at different temperatures, gives the following table:

Limit	Type of particle	n
$T >> m, \mu$		
	Bosons	$\frac{g\zeta(3)}{\pi^2}T^3$
	Fermions	$\frac{3g\zeta(3)}{4\pi^2}T^3$
$T \ll m$		
	Bosons	$g\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{(E-\mu)}{T}}$
	Fermions	$g\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{(E-\mu)}{T}}$

Table 1: Thermodynamics

The photons are known to have zero chemical potential (as the number of photons is not conserved), therefore the particle-antiparticle annihilation process: $A + \bar{A} \leftrightarrow \gamma + \gamma$ must satisfy the following conservation law: $\mu_A + \mu_{\bar{A}} = 0$.

Plugging this in the particle density equation, we find an asymmetry in the number of particles and antiparticles:

$$n_A - n_{\bar{A}} \cong \frac{g_A T^3}{6\pi^2}\left[\pi^2\left(\frac{\mu_A}{T}\right) + \left(\frac{\mu_A}{T}\right)^3\right]$$

with $T >> m_A$.

Note: at low temperature ($T << m_A$) we get, as expected, an exponential suppression of this asymmetry that goes like $e^{-\frac{m_A}{T}}$.

This asymmetry explains the domination of matter over antimatter and therefore allows the formation of nuclei.

The early universe being dominated by radiation, we can rewrite, using [table 1] the expansion rate of expansion of the universe:

$$H^2 = \frac{8\pi G}{3}\frac{\pi^2}{30}g_* T^4$$

H being homogenous to an inverse time, we can rewrite it as:

$$\boxed{t(T) = \frac{2.42}{T^2}\sqrt{g_*}}$$

with T in MeV, g_* being the number of relativistic degrees of freedom at a given temperature. The synopsis of the Big Bang nucleosynthesis can be split into three major phases:

T >> 1 MeV	Thermodynamic equilibrium between all the components of the universe ; Universes dominated by radiation ; $\frac{\#\boldsymbol{neutrons}}{\#\boldsymbol{protons}} \sim \mathbf{1}$ Photodissociation prevents any complex nuclei to form.
1>T >0.7Mev	Weak interaction cannot maintain equilibrium between all particles; neutrons decouple: “neutron freeze-out”: $\frac{\#\boldsymbol{neutrons}}{\#\boldsymbol{protons}} \sim \frac{\mathbf{1}}{\mathbf{6}}$. Free neutrons decay into protons; atomic nuclei stay at thermodynamic equilibrium. Freeze-out temperature T_f.
0.7>T>0.05Mev	Nuclear thermodynamic equilibrium cannot be maintained. Electron-positron annihilation has heated the photon bath. Atomic nuclei form through 2-body collisions: $p + n \rightarrow D + \gamma$ (only one way because radiation density is low enough).

Table 2: Thermal evolution of the universe

Big Bang Nucleosynthesis

a) Nuclear equilibrium

For temperatures greater or of the order of 100Mev, the universe is dominated by relativistic particles in equilibrium: electrons, positrons, neutrinos and photons. The contribution from non-relativistic particle can be neglected; the weak interactions between neutrons, protons and leptons:

$$\nu_e + n \leftrightarrow p + e$$

$$e^+ + n \leftrightarrow p + \bar{\nu_e}$$

$$n \leftrightarrow p + e + \bar{\nu_e}$$

and the electromagnetic interaction between electrons and positrons:

$$e + e^+ \leftrightarrow \gamma + \gamma$$

keep all the particles (as well as the non-relativistic baryons) in thermodynamic equilibrium.

At temperatures larger than 1Mev, the nuclear interactions still maintain the very first nuclei in thermodynamic equilibrium – their fraction can therefore be computed only using thermodynamic considerations, as shown below.
The density of those non-relativistic nuclei is therefore given by:

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{(m_A - \mu_A)}{T}} = g_A \left(\frac{m_A T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_A}{T}} (e^{\frac{\mu_p}{T}})^Z (e^{\frac{\mu_n}{T}})^{A-Z}$$

with μ_p, μ_n the chemical potential of protons and neutrons.
As long as the reaction rates are higher the expansion rate, the chemical equilibrium imposes the chemical potential to be:

$$\mu_A = Z\mu_p + (A - Z)\mu_n$$

Neglecting the difference in mass between protons and neutrons in the prefactor, we can extract from this formula the density ratio of protons and neutrons:

$$n_{p/n} = 2\left[\frac{(\frac{m_p + m_n}{2})\, T}{2\pi}\right]^{3/2} e^{-\frac{(m_{p/n} - \mu_{p/n})}{T}}$$

Rewriting the exponential in the nuclear density, we get:

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_A}{T}} = g_A \left(\frac{m_A T}{2\pi}\right)^{\frac{3}{2}} 2^{-A} n_p^Z n_n^{A-Z} \left(\frac{\pi}{m_N T}\right)^{3/2} e^{\frac{B_A}{T}}$$

With $B_A = Zm_p + (A - Z)m_n - m_A$ is the binding energy of the nucleon.

Defining $\eta = \frac{n_b}{n_\gamma}$, $X_A = \frac{A n_A}{n_N}$, $n_b = n_n + n_p + \sum A n_A$, we finally obtain the abundances of atomic nuclei in thermodynamic equilibrium:

$$\boxed{X_A = g_A A^{5/2} \left[\frac{\zeta(3)}{\pi} 2^{\frac{(3A-5)}{(2A-2)}}\right]^{A-1} \left(\frac{T}{m_N}\right)^{3(A-1)/2} \eta^{A-1} X_p^Z X_n^{A-Z} e^{\frac{B_A}{T}}}$$

with $\boxed{\eta = \frac{n_b}{n_\gamma}}$ the baryon to photon ratio.

We can use this formula to get the binding energy of the light nuclei and the temperature at which their abundance will be at a maximum:

	^2H	^3H	^3He	^4He
$B_A\ (MeV)$	2.22	6.92	7.72	28.3
$T_A\ (MeV)$	0.066	0.1	0.11	0.28

In particular, it gives the exact good primordial evolution for the Deuterium abundance up to its peak, and for the other light until elements until they depart from equilibrium (we'll discuss this point later).
From this table we understand that even at thermal equilibrium, nucleosynthesis cannot start before T=0.3MeV. Thermal photons prevent formation of large quantity of deuterium until T>0.3MeV (photodissociation) even though the cross section for $n + p \rightarrow d + \gamma$ is high.

b) Weak interaction freeze-out and neutron abundance

At some stage $t = t_f$, X can be said to freeze-out if its abundance stops evolving greatly.
Weak interaction reaction rate for $p + e \rightarrow v_e + n$ keeps protons and neutrons in equilibrium:

$$\Gamma_{\mathrm{weak}} = \frac{7\pi}{60}(1 + 3{g_A}^2){G_F}^2T^5$$

But when this reaction rate becomes of the order of the expansion rate, the thermal equilibrium is broken :

$$\frac{\Gamma}{H} = (\frac{T}{0.8})^3 \implies T_{\mathrm{freeze}} = 0.8\mathrm{MeV} \text{ or } t_{\mathrm{freeze}} = 1.15\mathrm{s}$$

Therefore, when the weak interaction (i.e. the neutron abundance) freezes out, the neutron to proton ration is:

$$X_{neutrons}(T_{\mathrm{freeze}}) = \frac{1}{1 + e^{\frac{m_n - m_p}{T}}} = 1/6$$

Just considering the expansion to get the neutron abundance at the beginning of nucleosynthesis is a bit too naïve though,), as the free neutron is not a stable particle it will decay with a lifetime of 887s from the time weak interaction freezes out to T_{nuc}, temperature that allows Deuterium to form (T=0.086MeV, t=180sec). We therefore get :

$$\boxed{X_{neutrons}(T_{nuc}) = \frac{1}{6} e^{-t/\tau} = 0.136}$$

c) Element synthesis: Primordial abundances of ^4He, ^3He, Deuterium, ^7Li

Let's start our analysis with qualitative arguments:
The formation of deuterium is a crucial factor for the continuation of the Big Bang nucleosynthesis: if too much ^2H is formed, then neutrons are locked up and no heavier elements can be created. However if too little is formed, then an important factor in any further fusion is missing; especially, when deuterium can form, helium4 can form much more easily because the following reactions $d + d \rightarrow {}^3H + p$ and $d + d \rightarrow {}^3He + n$ enable the creation of large amounts for Helium through $^3H + d \rightarrow {}^4He + n$ and $^3He + d \rightarrow {}^4He + p$.

A very important factor in the Big Bang nucleosynthesis is that no stable elements exist with A=5 and A=8 (figure 4). Therefore we would expect this "anomaly" to greatly decrease any nucleation process at A=4 and A=7 (it is a lot less likely for A=4 to go to A=6 than to A=5).
From this, we understand that:

- The abundance of ^2H should at first increase then decrease when Helium-4 can be produced (as it is being consumed in the reactions).
- The abundances of ^3H and ^3He should increase then level up or decrease when helium is getting created. They should have a lower final abundance than Helium-4 (tritium is radioactive therefore should decrease over long periods –longer than the one studied here)
- Helium-4, which is stable, and at the end of the "first chain" (before A=5) should be present in large quantities at the end of the Big Bang nucleosynthesis. All the previous elements can fuse to form Helium-4, and Helium-4 is very unlikely (but not 0) to react with protons or neutrons to jump to A=6
- Abundances of Lithium and Beryllium should be very low compared to the other. We can expect ^7Be and ^7Li to be the end of the Big Bang nucleosynthesis because of the A=8 "barrier" (also helped by the fact that Be7 is radioactive and therefore decays to lighter elements).
- Lithium valley: destroyed by protons but Be7 contribution.

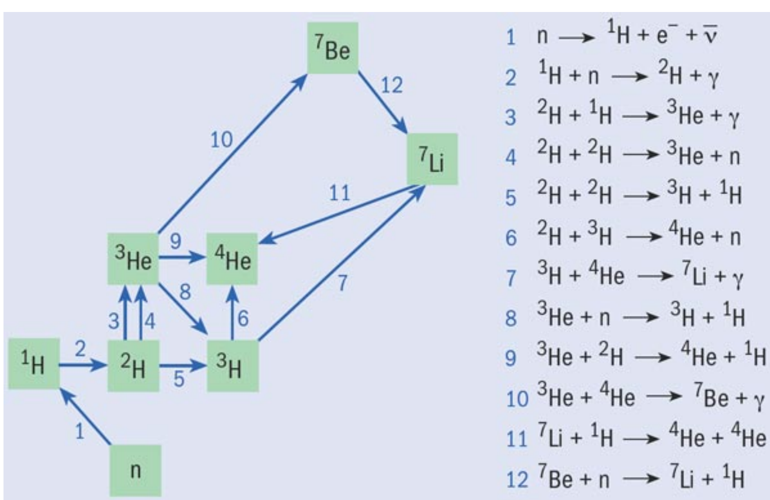


Figure 1: Nuclear reactions - http://physicsworld.com/cws/article/print/30680/1/PWfea4_08-07

Now let's get into some little computations:
The easiest estimation is for Helium-4: we now that all free neutrons left after freeze-out will get bound up to He-4 because of nuclei stability. Estimations on He-4 abundances depend mostly on the neutron to proton rate, thus most of the uncertainties are based on the uncertainties of the free neutron lifetime (τ= 887 seconds)

$$\boxed{X_{He4} \sim \frac{2 * X_{neutrons}(T_{nuc})}{1 + X_{neutrons}(T_{nuc})} = 0.24}$$

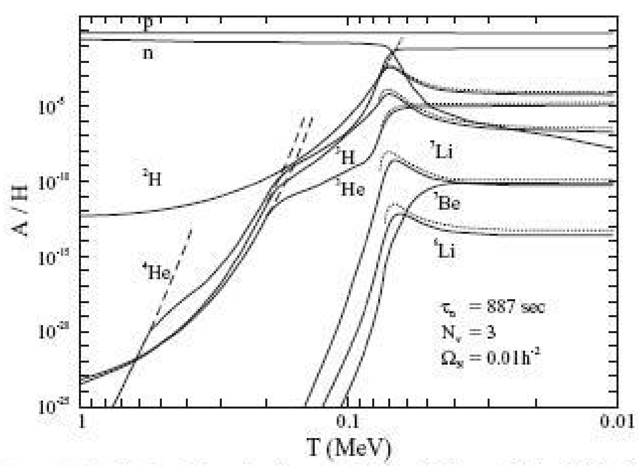
For Deuterium, we can get its creation slope with the thermal-equilibrium equation given previously (simply using Mathematica, it work perfectly); however getting its final abundance is much more difficult as we have to compute all its "sink terms". The same problem appears for all the other elements.
It is however doable without a computer, starting with the very general statement:

$$\frac{dX}{dt} = J(t) - \Gamma(t)X$$

where J(t) and $\Gamma(t)$ are time-dependant source and sink terms.
The solution to this equation is

$$X(t) = e^{-\int_{t_i}^{t} dt'\Gamma(t')}[X(t_i) + \int_{t_i}^{t} dt' J(t')\, e^{-\int_{t_i}^{t} dt''\Gamma(t'')}]$$

This requires, as expected, careful examination of all the different reaction networks and keeping track of their reaction rate as temperature decreases, therefore it is a very tedious computation (figure 2)[1].
Generally freeze-out occurs when $\Gamma \sim H$. It is possible to show that X approaches this equilibrium value for $X_{t\rightarrow\infty} = X_{eq}(t_f) = \frac{J(t_f)}{\Gamma(t_f)}$



Evolution of the abundances of primordially synthesized light elements with temperature according to the Wagoner (1973) numerical code as upgraded by Kawano (1992). The dashed lines show the values in nuclear statistical equilibrium while the dotted lines are the 'freeze-out' values as calculated analytically by Esmailzadeh *et al* (1991).

Figure 2: Analytic computation
http://arxiv.org/PS_cache/hep-ph/pdf/9602/9602260v2.pdf

[1] Esmailzadeth *et al* (1991) ; Dimopoulos *et al* (1988)

The analytical results shown here give the good predictions for D, Helium-3 and Lithium-7 within a factor of 3, and Helium-4 within 5%!
Moreover, this analytic calculation helps in understanding the depart of Helium-4 from its nuclear statistical equilibrium (the dashed line around 0.6Mev).
This behavior is explained by the “deuterium bottleneck” (figure 1): the creation of Helium-4 is delayed until enough tritium and helium-3 are formed (it then mimics the evolution of T and Helium-3).
Then they too depart from nuclear statistical equilibrium at about 0.2MeV due to the “deuterium bottleneck”. At this point Helium-4, Helium-3 and Tritium all mimic the evolution of Deuterium until it finally deviates from equilibrium at 0.07MeV.

We can now compare theory and the observation (figure 4) on the final abundances of the light elements:

	Theory	Observation
D	$3.6 * 10^{-5\pm0.06}\left(\frac{\eta}{5.5 * 10^{-10}}\right)^{-1.6}$	$2.78 \pm 0.44 * 10^{-5}$ $> 1.5 \pm 0.1 * 10^{-5}$ $< 6.7 * 10^{-5}$
^3He	$1.2 * 10^{-5\pm0.06}\left(\frac{\eta}{5.5 * 10^{-10}}\right)^{-0.63}$	$1.5 \pm 0.5 * 10^{-5}$
^4He	$0.245 \pm 0.014(N_\nu - 3) \pm 0.0002(\tau_n - 887) \pm 0.009 \ln\left(\frac{\eta}{5.5*10^{-10}}\right)$	0.2443 ± 0.0015 0.2391 ± 0.0020 0.249 ± 0.004
^7Li	$1.2 * 10^{-10\pm0.06}\left(\frac{\eta}{5.5 * 10^{-10}}\right)^{-2.38}$	$1.23^{+0.68}_{-0.32} * 10^{-10}$

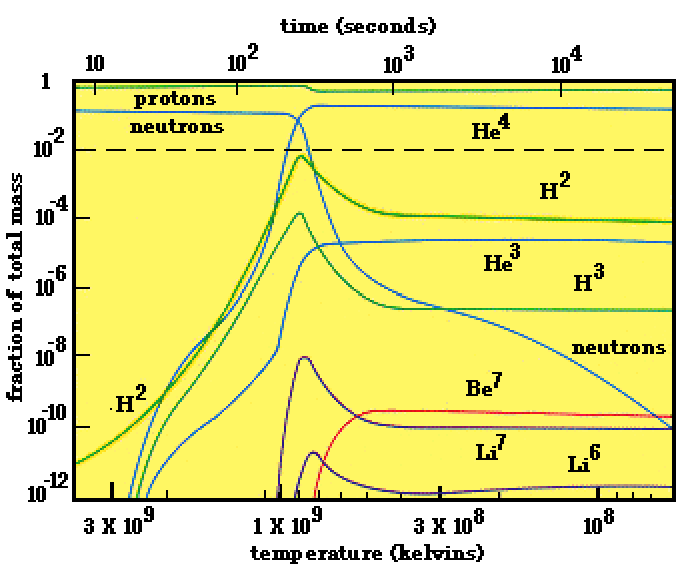


Figure 3: Abundances
http://www.astro.ucla.edu/~wright/BBNS.html

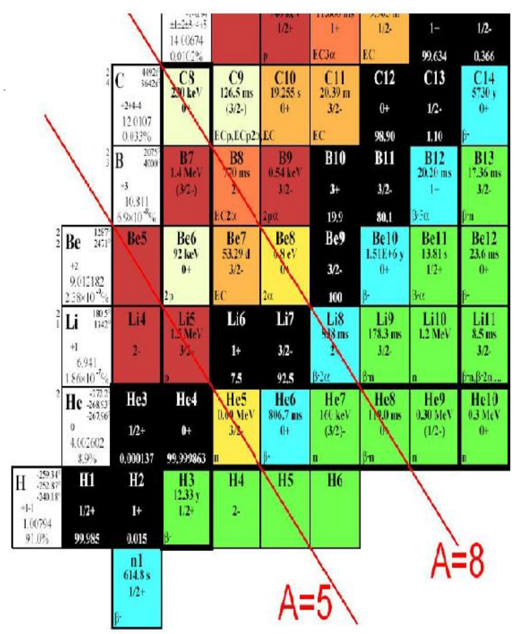


Figure 4: Element gap
“Big Bang Nucleosynthesis C. Ciemniak 14.05.2004”

d) Uncertainties

As nucleosynthesis involves many different reactions, there are many sources of uncertainty. The major uncertainties in ^{4}He are due to

- The experimental uncertainty on the neutron lifetime. The current value is considered to be[2] $\tau_{\upsilon} = 887 \pm 2\ s$. A change of 2σ in the neutron lifetime causes a change in Helium-4 prediction of about 0.4%.
- The number of neutrino generations ; if the number of generations is more than 3, say 3.3 (as we will this that seems to be the current bound on the number of neutrino generations) then we get a change of about 1.5% in the abundance prediction.

For the other elements, uncertainties in the nuclear cross sections can dramatically modify the abundance predictions. They can alter D and ^{3}He by up to 15% and ^{7}Li by about 50%!
And all of this being a “chain reaction” a change in the reaction rate of one element will affect all the others.
These effects can be neglected for the Helium-4 abundance ($\leq 0.3\%$) because as we have seen it is possible to calculate its abundance only using the neutron’s abundance at nucleation time; we did not have to consider any of the nuclear reactions to do this calculations and already got a very precise result.

Big Bang nucleosynthesis also allows to calculate the baryon to photon ratio η.
From spectroscopic measurements, and taking $N_{\nu} = 3$, one gets the following value:

$$\eta = 5.15 \pm 1.75 * 10^{-10}$$

Which corresponds to $\Omega_{\mathrm{B}}\mathrm{h}^2 = 0.018 \pm 0.006$ where Ω_{B} is the baryonic content of the universe, and h the reduced Hubble constant.
As we will see later those parameters allow us to put constraints on other observables like the number of neutrinos generations, their masses, but also to test the value of the fundamental constants like G and the fine structure constant α (constraint given by BBN is $\frac{\Delta\alpha}{\alpha} < 5\%$.)

Big Bang nucleosynthesis is therefore a test for the hot Big Bang model, nuclear physics, and astrophysics in general.

[2] Particle Data Group

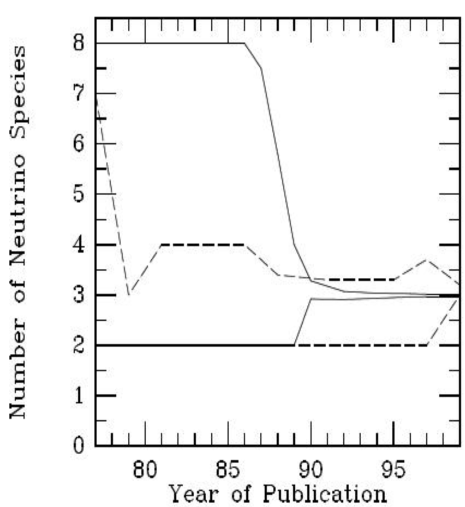
Constraints on new physics

a) Number of neutrino generations

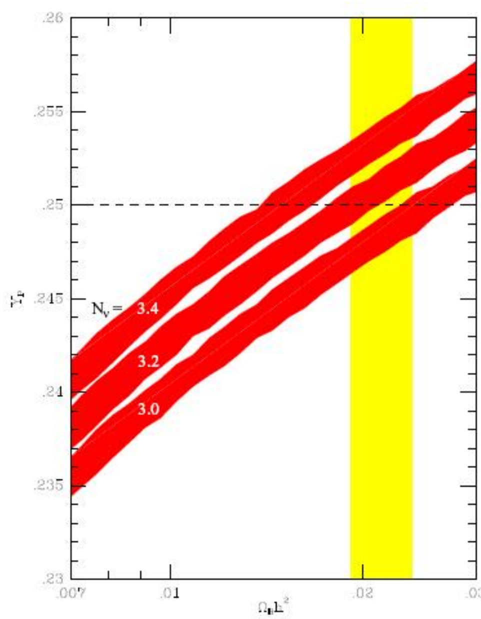
An increase in the number of neutrino N_ν leads to an increase in the expansion rate of the universe, therefore more neutrons will survive until nucleosynthesis which leads to an increase in the Helium-4 abundance.

Number of Neutrino Flavors	Model predictions for He-4
$N_\nu = 2$	$Y \sim 0.227$
$N_\nu = 3$	$Y \sim 0.242$
$N_\nu = 4$	$Y \sim 0.254$

Constraints from Big Bang nucleosynthesis are still very difficult to estimate (many different values can be found in the literature). It seems like the best fit $\boxed{N_\nu \sim 3 \pm 0.3}$.



Cosmological (broken curve) and laboratory (solid curve) limits (95% cl) to the number of neutrino species. An ultimate cosmological limit of 3.1 neutrino species has been "anticipated."



^{4}He production for $N_\nu = 3.0, 3.2, 3.4$. The vertical band indicates the baryon density consistent with $(D/H)_P = (2.7 \pm 0.6) \times 10^{-5}$ and the horizontal line indicates a primeval ^{4}He abundance of 25%. The widths of the curves indicate the two-sigma theoretical uncertainty.

Credits : arXiv:astro-ph/9706069

b) Neutrino masses

Although in the "naïve" version of the Standard Model neutrinos are massless, recent experiments have tended to show that neutrinos were actually massive (from neutrino oscillation experiments mainly[3]).

Even though their mass is thought to be extremely small, the large quantity of neutrinos implies that any non-zero mass have an observable impact on cosmology.

From nucleosynthesis, we can get an upper limit to the mass of all the neutrinos, by combining the relic neutrino abundance with observational bounds on present energy density (thanks to WMAP):

$$\sum_i m_{\upsilon_i}\left(\frac{g_{\upsilon_i}}{2}\right) \leq 94\ eV$$

summing over all neutrino species that are relativistic at decoupling (i.e. $m_{\upsilon_i} \leq 1MeV$).

If we consider that neutrinos are more massive and therefore where not relativistic (!), then they fall out of chemical equilibrium before the relativistic decoupling, the we get a lower limit on their mass: $m_{\upsilon_i} \geq 2GeV$.

Therefore no stable neutrino can have a mass in the 100eV to 2GeV range.
Current neutrino mass estimations are:

$$m_{\upsilon_e} \leq 2.1eV \quad , \quad m_{\upsilon_\mu} \leq 160keV \quad , \quad m_{\upsilon_\tau} \leq 24MeV$$

That is for now, only the electron-neutrino is known to have a mass out of the forbidden region.

c) Dark matter

Direct observations of luminous matter give the following result toward the content of the universe:

$$(\Omega \mathrm{h}^2)_{\mathrm{vis}} \sim 0.005$$

Comparing this to the previous value of $\Omega_{\mathrm{B}}\mathrm{h}^2$, we quickly understand that there is a problem : $\frac{(\Omega\mathrm{h}^2)_{\mathrm{vis}}}{\Omega_{\mathrm{B}}\mathrm{h}^2} = 30\%$! Where are the other 70%?

- Baryonic => non nucleonic dark matter, planetary mass black holes, "strange quark nuggets" (should have enhanced production of heavy elements during BBN).
- Non-baryonic => Relic particles? New particles?

[3] Super-Kamiokande, 1998

Conclusion

For a long time, primordial nucleosynthesis and spectroscopic measurements where the only way to predict and test the light elements abundances, and to get a value for the baryonic content of the universe. However recent calculations[4] of the Cosmic Microwave Background (CMB) have allowed to measure with extreme accuracy the baryonic content of the universe:

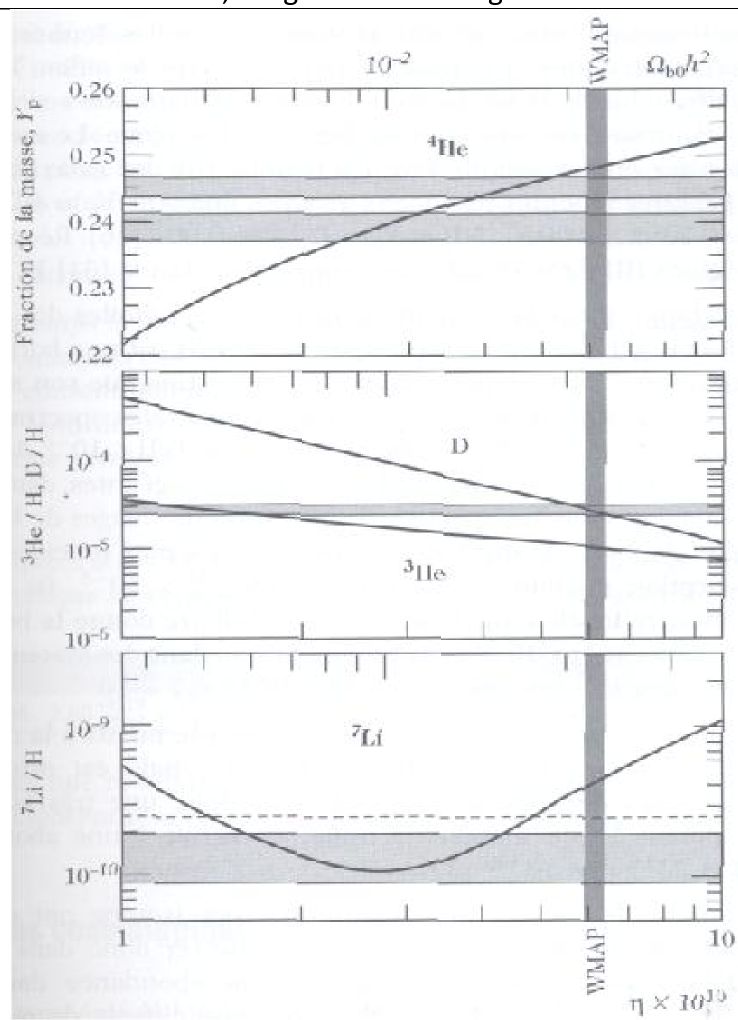
$$\Omega_b h^2 = 0.0224 \pm 0.0009$$

which corresponds to a baryon to photon ratio :

$$\boxed{\eta = 6.14 \pm 0.25 * 10^{-10}}$$

If we redo all the previous calculations with those values, we get the following abundances:

Element	Theoretical prediction using CMB values
D	$2.60^{+0.19}_{-0.17} * 10^{-5}$
^3He	$1.04^{+0.04}_{-0.04} * 10^{-5}$
^4He	0.247 ± 0.0004
^7Li	$4.15^{+0.49}_{-0.45} * 10^{-10}$



On this graph are shown the theoretical predictions for Deuterium, Helium-3, Helium-4 and Lithium-7 with respect to the baryon photon ratio.

The vertical line represents WMAP value for η, the horizontal ones indicate the spectroscopic measurements for each element, the width representing the uncertainties.

The curves are the BBN predictions.
Credits : “Cosmologie Pirmordiale”
Uzan & Peter

Those results are in conflict with the theoretical predictions. Nuclear reaction rates are being reanalyzed because, as we have seen, a slight change in one of them could greatly affect all the predictions.

[4] WMAP results on the CMB

References

Books

"Cosmologie Primordiale" – Jean Philippe Uzan & Patrick Peter

"Principles of physical cosmology" P.J.E Peebles

ArXiV

arXiv:hep-ph/9602260 : Big Bang nucleosynthesis and physics beyond the Standard Model

arXiv:astro-ph/9706069 : Big-bang Nucleosynthesis Enters the Precision Era

arXiv:astro-ph/0009506 : Light Element Nucleosynthesis

arXiv:astro-ph/9905211 : Primordial Lithium and Big Bang Nucleosynthesis

arXiv:astro-ph/0008495 : What Is The BBN Prediction for the Baryon Density and How Reliable Is It?

arXiv:astro-ph/9905320 : Primordial Nucleosynthesis: Theory and Observations

Other Internet resources

http://www.astro.ucla.edu/~wright/BBNS.html
http://www-thphys.physics.ox.ac.uk/users/SubirSarkar/mytalks/groningen05.pdf
http://www.astro.uu.se/~bg/cosmology.pdf
http://www.mpia.de/homes/rix/BBN_Lect.pdf
Mark Trodden and Sean Carroll "TASI Lectures: Introduction to Cosmology"