Introduction to Relativity

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This document is under construction

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The electric field $\overrightarrow{E}$ has 3 components. The magnetic field $\overrightarrow{B}$ has 3 components.

So we need to create an object that has 6 independant components => Antisymmetric 4x4 tensor : number of free components = $\frac{n(n-1)}{2} = 6$

It's a second rank tensor that transforms as follows :

$F^{\mu\nu} = \Lambda^{\mu}_{\lambda}\Lambda^{\nu}_{\sigma}F^{\lambda\sigma}$

Electromagnetic tensor (with c=1) :

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & B_z & -B_y \\ -E_y & -B_z & 0 & B_x \\ -E_z & B_y & -B_x & 0 \end{pmatrix}$$

Its dual tensor is :

$$\overline{F}^{\mu\nu} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{pmatrix}$$

Computing $\partial_\mu F^{\mu\nu}$ and $\partial_\mu \overline{F}^{\mu\nu}$ leads to all Maxwell equations.

Lorentz force :

$f^{\mu} = qu_{\nu}F^{\mu\nu} = ma^{\mu}$

$f = \frac{q}{\sqrt{1-\frac{\overrightarrow{u}^2}{c^2}}}[\overrightarrow{E} + (\overrightarrow{u} \times \overrightarrow{B})]$

Spatial component of f :

$f = \frac{F_{Lorentz}}{\sqrt{1-\frac{\overrightarrow{u}^2}{c^2}}}$

Therefore :

$\overrightarrow{F}_{Lorentz} = q[\overrightarrow{E} + (\overrightarrow{u} \times \overrightarrow{B})]$

1.3 Spacetime and Relativistic dynamics

1.3.1 Minkowski spacetime

1.3.2 Equations of motion

1.3.3 Applications of SR : High energy physics and paradoxes

2 General Relativity

Tidal forces in the framework of general relativity Weak, time independant field approximation

Metric : $ds^2 = (1-2\phi)dt^2 - (1-2\phi)(dx^2+dy^2+dz^2)$

so $ds^2 = g_{ij}dx^i dx^j$ with

$$g_{ij} = \begin{bmatrix} 1-2\phi & 0 & 0 & 0 \\ 0 & -(1-2\phi) & 0 & 0 \\ 0 & 0 & -(1-2\phi) & 0 \\ 0 & 0 & 0 & -(1-2\phi) \end{bmatrix}$$

Hydrostatic equilibrium (spring tides) :

$0 = g_{R_e+h} + R_e R^{Sun}_{2020} + R_e R^{Moon}_{2020}$

$0 = g_{R_e-h} + R_e R^{Sun}_{3030} + R_e R^{Moon}_{3030}$

Substracting, and expanding g in series we get :

$2hg'_{R_e} + R_e(R^{Sun}_{2020} - R^{Sun}_{3030} + R^{Moon}_{2020} - R^{Moon}_{3030}) = 0$

with :

$R^{Sun}_{2020} = \frac{GM_{sun}}{d^3_{sun}}$; $R^{Sun}_{3030} = -2\frac{GM_{sun}}{d^3_{sun}}$

$R^{Moon}_{2020} = \frac{GM_{moon}}{d^3_{moon}}$; $R^{Moon}_{3030} = -2\frac{GM_{moon}}{d^3_{moon}}$

Therefore we have :

$2hg'_{R_e} + 3R_e(\frac{GM_{sun}}{d^3_{sun}} + \frac{GM_{moon}}{d^3_{moon}}) = 0$

But $g\prime = -2\frac{GM_{earth}}{R^3_e}$

so we get :

$-4h\frac{GM_{earth}}{R^3_e} + 3R_e(\frac{GM_{sun}}{d^3_{sun}} + \frac{GM_{moon}}{d^3_{moon}}) = 0$

and finally :

$h = \frac{3}{4}\frac{R^4_e}{M_{earth}}(\frac{M_{sun}}{d^3_{sun}} + \frac{M_{moon}}{d^3_{moon}})$

Numerical result :

$R_e = 6400km$

$M_{earth} = 6 * 10^{24}kg$

$M_{Moon} = 7.34 * 10^{22}kg$

$M_{Sun} = 2 * 10^{30}kg$

$d_{Moon} = 3.84 * 10^8 m$

$d_{Sun} = 1.5 * 10^{11} m$

We get $h \approx 40cm$